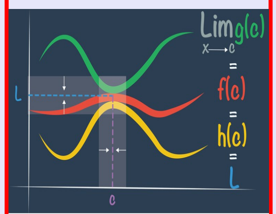
# Math 261

# Fall 2022

# Lecture 41




Feb 19-8:47 AM

More on Substitution Method:

Find $\int_0^4 \sqrt{2x+1}\, dx = \int_1^9 \sqrt{u} \cdot \frac{du}{2}$

Let $\boxed{u = 2x+1}$

$x=0 \to u=1$

$x=4 \to u=9$

$\frac{du}{dx} = 2 \qquad du = 2dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2}\int_1^9 u^{1/2}\, du$$

$$= \frac{1}{\cancel{2}} \cdot \frac{u^{3/2}}{3/\cancel{2}} \Big|_{u=1}^{u=9}$$

$$= \frac{1}{3} \cdot u\sqrt{u} \Big|_1^9$$

$$= \frac{1}{3}\left[9\sqrt{9} - 1\sqrt{1}\right] = \frac{1}{3}[27-1] = \boxed{\frac{26}{3}}$$

Nov 9-8:49 AM

Evaluate $\frac{1}{2}\int_0^{\sqrt{\pi}} 2x \cos x^2 \, dx$

Let $u = x^2$

$x = 0 \rightarrow u = 0$

$x = \sqrt{\pi} \rightarrow u = \pi$

$du = 2x \, dx$

$= \frac{1}{2}\int_0^{\pi} \cos u \, du$

$= \frac{1}{2} \sin u \Big|_0^{\pi} = \frac{1}{2}\left[\sin \pi - \sin 0\right]$

$= \frac{1}{2}\left[0 - 0\right] = \boxed{0}$

Nov 9-8:54 AM

Find $\int (2x+1) \sec^2(x^2+x) \, dx$

$u = x^2 + x$

$du = (2x+1)\, dx$

$= \int \sec^2 u \, du = \tan u + C$

$= \tan(x^2+x) + C$

Nov 9-8:59 AM

Evaluate $\int_0^{\sqrt{3}} \frac{x}{\sqrt{1+x^2}}\,dx = \int_0^{\sqrt{3}} x\,(1+x^2)^{-1/2}\,dx$

$\hookrightarrow u = 1+x^2$

$x=0 \to u=1$

$x=\sqrt{3} \to u=4$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int_1^4 u^{-1/2} \cdot \frac{du}{2}$

$= \frac{1}{2}\int_1^4 u^{-1/2}\,du = \frac{1}{\cancel{2}} \cdot \frac{u^{1/2}}{\cancel{1/2}} \Big|_{u=1}^{u=4} = \sqrt{u}\,\Big|_1^4 = \sqrt{4} - \sqrt{1}$

$= \boxed{1}$

Nov 9-9:03 AM

find $\int_0^8 (2x-8)\sec(x^2-8x)\tan(x^2-8x)\,dx$

$u = x^2-8x$

$du = (2x-8)\,dx$

$\int_0^0 \sec u \tan u\,du$

$x=0 \to u=0$

$x=8 \to u=0$

$= \sec u \Big|_0^0 = \sec 0 - \sec 0 = \boxed{0}$

Rule $\int_a^a f(x)\,dx = 0$ $\qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Nov 9-9:09 AM

Find $\int \sqrt[3]{1+x^2} \cdot x^3 \, dx = \int \sqrt[3]{1+x^2} \cdot \boxed{x} \cdot x^2 \boxed{dx}$

Notice $x^3 = x \cdot x^2$

Let $u = 1 + x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int \sqrt[3]{1+x^2} \cdot (u-1) \frac{du}{2}$

$= \int \sqrt[3]{u}\,(u-1) \frac{du}{2}$

$= \frac{1}{2} \int \left[ u^{4/3} - u^{1/3} \right] du$

$= \frac{1}{2} \left[ \frac{u^{7/3}}{7/3} - \frac{u^{4/3}}{4/3} \right] + C$

$= \frac{1}{2} \left[ \frac{3}{7} u^{7/3} - \frac{3}{4} u^{4/3} \right] + C$

$= \frac{3}{14} \sqrt[3]{u^7} - \frac{3}{8} \sqrt[3]{u^4} + C$

$= \frac{3}{14} \sqrt[3]{u^6}\sqrt[3]{u} - \frac{3}{8} \sqrt[3]{u^3}\sqrt[3]{u} + C$

$= \frac{3}{14} u^2 \sqrt[3]{u} - \frac{3}{8} u \sqrt[3]{u} + C$

$= \boxed{\frac{3}{14} (1+x^2)^2 \sqrt[3]{1+x^2} - \frac{3}{8} (1+x^2) \sqrt[3]{1+x^2} + C}$

Nov 9-9:16 AM

Subs. Rule:

If . . - - - - - -

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

where $u = g(x)$

Nov 9-9:34 AM

find $\int_0^{\pi} \sec^2\left(\frac{x}{4}\right) dx$

$u = \frac{x}{4}$ $\quad x=0 \to u=0$

$4u = x$ $\quad x=\pi \to u=\frac{\pi}{4}$

$4\,du = dx$

$$\int_0^{\pi/4} \sec^2 u \cdot 4\,du$$

$$= 4\int_0^{\pi/4} \sec^2 u\,du = 4\cdot \tan u\Big|_0^{\pi/4} = \boxed{4}$$

Nov 9-9:36 AM

Evaluate $\int_1^2 \frac{\cos\left(\frac{\pi}{x}\right)}{x^2} dx$

$u = \frac{\pi}{x}$

$du = \frac{-\pi}{x^2} dx$

$\frac{du}{-\pi} = \frac{1}{x^2} dx$

$x=1 \to u=\pi$

$x=2 \to u=\frac{\pi}{2}$

$\int_b^a f(x)dx = -\int_a^b f(x)dx$

$$= \int_{\pi}^{\pi/2} \cos u \frac{du}{-\pi}$$

$$= \frac{-1}{\pi}\int_{\pi}^{\pi/2} \cos u\,du$$

$$= \frac{1}{\pi}\int_{\pi/2}^{\pi} \cos u\,du = \frac{1}{\pi}\left[\sin u\Big|_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\cancelto{0}{\sin\pi} - \cancelto{1}{\sin\frac{\pi}{2}}\right] = \boxed{\frac{-1}{\pi}}$$

Nov 9-9:39 AM

Evaluate $\int_0^{\pi} \sin^3 x \cos x \, dx$

$x=0 \to u=0$

$x=\pi \to u=0$

$u=\sin x$

$du=\cos x \, dx$

$$= \int_0^0 u^3 \, du = \boxed{0}$$

Pick $u=\cos x$ $\quad x=0 \;\; u=1$, $\quad x=\pi \;\; u=-1$

$du=-\sin x \, dx$

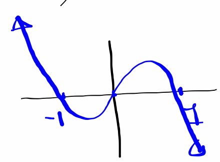
$$\int \sin^3 x \cos x \, dx$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1-\cos^2 x$

$$= \int \sin^2 x \cdot \cos x \cdot \sin x \, dx$$

$$= \int_1^{-1} (1-u^2) \cdot u \cdot -du = \int_{-1}^{1} (u-u^3)\, du = 0$$




Nov 9-9:46 AM

Find $\int \sqrt[3]{1+x^2} \cdot x^3 \, dx$ $\quad \to x \cdot x^2$

Let $u=\sqrt[3]{1+x^2}$ $\quad u^3 = 1+x^2$

$3u^2 du = 2x \, dx$

$$= \int u \cdot x^2 \cdot (x \, dx)$$

$$= \int u (u^3-1) \frac{3u^2}{2} du = \frac{3}{2} \int u^3 (u^3-1)\, du$$

$$= \frac{3}{2} \int (u^6 - u^3)\, du$$

$$= \frac{3}{2} \left[ \frac{u^7}{7} - \frac{u^4}{4} \right] + C$$

$$= \frac{3}{14} u^7 - \frac{3}{8} u^4 + C$$

$$= \frac{3}{14} \left(\sqrt[3]{1+x^2}\right)^7 - \frac{3}{8} \left(\sqrt[3]{1+x^2}\right)^4 + C$$

Nov 9-9:29 AM